Flows of a Vapor due to Phase Change Processes at the Condensed Phases with Temperature Fields as their Internal Structures

Yoshimoto Onishi and OOSHIDA Takeshi

Department of Applied Mathematics and Physics, Tottori University, Tottori 680-8552, Japan

Abstract. Transient to steady motions of a vapor caused by the evaporation and condensation processes occurring at the condensed phases placed in parallel have been studied based on the Boltzmann equation of BGK type. As the internal structures of the condensed phases, the temperature fields are taken into account. Because of this, the temperatures of the interfaces become unknown parameters and, therefore, the condition of the continuity of energy flow across the interface has to be imposed simultaneously with the conditions so far used for the cases with no internal structures. This extra condition gives great difficulty in the numerical simulations but this has been surmounted by a simple method developed earlier in our laboratory. The present analysis has also incorporated a certain kind of imperfectness of the interface in the boundary conditions by the introduction of a simple parameter, called the *imperfectness* parameter here, first proposed by Wortberg and his colleague. The results obtained describe appropriately the development of the transient flow fields due to the processes of evaporation and condensation at the interfaces across which the continuous energy flows are taking place. Some of the features worth to be mentioned are that 1) a certain value of the latent heat parameter gives the maxima of the mass and energy flows. This fact, which is newly found here, is due to the coupling effects of the latent heat parameter and the existence of the internal structures of the condensed phases; 2) the *negative temperature gradient* phenomenon, a well-known phenomenon at steady state in weak evaporation and condensation problems between the two condensed phases having no internal structures, seems to be non-existent in the present case with internal structures; 3) the *negative mass flow* phenomenon, first noticed and discussed by Sone and Onishi, seems to be non-existent at steady state but this surely manifests itself in a short period of time during the transitional state of the flow fields.

INTRODUCTION

An extensive studies on flow problems associated with evaporation and condensation processes have been worked out so far because problems of this kind are not only of theoretical interest but also of practical importance, especially in various thermo-fluid engineering fields. Most of the analyses done so far, however, are confined to flow systems in which the condensed phases have no internal structures at all. The existence of the internal structure within the condensed phase may affect drastically the behavior of the gas phase and, hence, the mass and energy transfers. In addition, practically important heat and energy exchange systems or transfer systems such as heat pipes may be considered to be closely associated with such flow systems having internal structures.

Recently, Onishi and Yamada [1] have carried out a numerical analysis on the half-space problem based on the Boltzmann equation of BGK type [2], i.e., evaporation and condensation flows from or onto the plane condensed phase with temperature field as its internal structure, revealing the effects of the various parameters involved, the parameters associated with the thermal conductivity and thermal diffusivity of the condensed phase and also the latent heat of vaporization, on the flow fields. Among others, the role of the latent heat parameter was found to be totally different from the case in which the condensed phase has no temperature field, i.e., the condensed phase has the infinitely large thermal conductivity. To be specific, for a certain range of values of the latent heat parameter, the phase change processes become strong and, therefore, the associated mass and energy flows become large. This indicates that there may exist a certain value of the latent heat parameter which gives the maxima of the mass and energy flows. This fact is in distinctive contrast to what we know from the studies done so far about the mass and energy flows when the condensed phases have no temperature fields as their internal structures. In the latter case, the mass and energy flows become large with the values of the latent heat parameter. In view of this, theoretical and practical interests will surely be aroused as to the effects of those parameters mentioned above on the two-surface problem in which

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
13 JUL 2005	**N/A**	**-**

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Flows of a Vapor due to Phase Change Processes at the Condensed Phases with Temperature Fields as Their Internal Structures	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Department of Applied Mathematics and Physics, Tottori University, Japan	

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM001792, International Symposium on Rarefied Gas Dynamics (24th) Held in Monopoli (Bari), Italy on 10-16 July 2004. , The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**UU**	**6**	

two condensed phases are involved, i.e., on the flow fields confined by the two condensed phases having temperature fields as their internal structures (having finite thermal conductivities). This is what the present work is concerned with. The reason is that, in this two-surface problem, when the condensed phases have no internal structures at all, the phenomena of the so-called *negative temperature gradient* [3]-[4] and the *negative mass flow* [4] may appear. The former phenomenon manifests itself for values of the latent heat parameter larger than a certain definite value when the phase change processes are fairly weak. The latter does for its extremely smaller values.

Finally, a certain kind of imperfectness of the interfaces is also taken into account in the present study through the boundary conditions for the distribution functions at the interfaces. This is expressed in terms of a parameter, called the *imperfectness parameter* here, first introduced by Wortberg and his co-workers [5]. The effects of this parameter, however, will not be discussed here owing to the limited pages.

FORMULATION OF THE PROBLEM

Consider a vapor between the two condensed phases with finite thermal conductivity placed in parallel. The condensed phases occupy regions of $-D_1 \leq x \leq 0$ and $L \leq x \leq L + D_2$, respectively. Let the interface surfaces between the gas phase and the condensed phases be at $x = 0$ and $x = L$. Initially, the vapor phase and its condensed phases are in complete equilibrium at a temperature T_0. Let the pressure and density (or number density) of the vapor at this state be P_0 and ρ_0 (or N_0), respectively. Suppose that at a certain time, say $t = 0$, the temperatures of the edge surfaces of the condensed phases at $x = -D_1$ and $x = L + D_2$ are suddenly changed to T_{c1} and T_{c2}, respectively. The heat flows then occur through the condensed phases and after a certain elapse of time the temperatures of the interface surfaces start to be changing with time. This leads to the onset of phase change processes at the interfaces, giving then rise to a transient motion of the vapor in between. This is what the present study is concerned with

The equation of heat conduction with constant substance properties is assumed to be able to govern the temperature field within each of the condensed phases, which may be written as

$$\left.\begin{array}{ll} \dfrac{\partial \tilde{T}_1}{\partial t} - \kappa_c \dfrac{\partial^2 \tilde{T}_1}{\partial x^2} = 0 & (-D_1 < x < 0) \\[4mm] \dfrac{\partial \tilde{T}_2}{\partial t} - \kappa_c \dfrac{\partial^2 \tilde{T}_2}{\partial x^2} = 0 & (L < x < L + D_2) \end{array}\right\} \tag{1}$$

where t is the time; x is the coordinate; κ_c is the thermal diffusivity of the condensed phases and $\tilde{T}_i$ $(i = 1, 2)$ represents the temperature fields. For the description of the motions of the vapor in the present problem, on the other hand, the Boltzmann equation of BGK type [2] is used, which may be written as

$$\frac{\partial f}{\partial t} + \xi_x \frac{\partial f}{\partial x} = N \nu_c (F_e - f) \tag{2}$$

$$F_e = \frac{N}{(2\pi RT)^{3/2}} \exp\left\{-\frac{(\xi_x - u)^2 + \xi_y^2 + \xi_z^2}{2RT}\right\} \tag{3}$$

$$\begin{bmatrix} N \\ Nu \\ \frac{3}{2}NkT \end{bmatrix} = \iiint \begin{bmatrix} 1 \\ \xi_x \\ \frac{1}{2}m[(\xi_x - u)^2 + \xi_y^2 + \xi_z^2] \end{bmatrix} f \, d\xi_x d\xi_y d\xi_z \tag{4}$$

$$P = NkT = \rho RT \tag{5}$$

where (ξ_x, ξ_y, ξ_z) is the molecular velocity vector; f is the molecular velocity distribution function, F_e being the local Maxwellian distribution characterized by the local fluid dynamic quantities; N, u, T, P and ρ are, respectively, the number density, the velocity, the temperature, the pressure and the density of the gas; m is the molecular mass; k is the Boltzmann constant and $R = k/m$ the gas constant per unit mass of gas. ν_c is a constant associated with the collision frequency ($N\nu_c$ is the local collision frequency) and, hence, can be calculated either from the viscosity μ or from the thermal conductivity λ of the gas at a certain reference state, say, at the initial equilibrium state, by the following relation

$$N_0 \nu_c = \frac{P_0}{\mu_0} = \frac{5}{2} R \frac{P_0}{\lambda_0} \tag{6}$$

the suffix 0 being understood to indicate the quantities at the initial state. It may be noted here that, in the BGK model equation, the relation $\lambda = (5/2)R\mu$ holds and, hence, the Prandtl number Pr is unity for this model equation.

The initial conditions for the present problem may be specified as follows:

$$\tilde{T}_1 = \tilde{T}_2 = T_0 \qquad (-D_1 \leqq x \leqq 0, \quad L \leqq x \leqq L + D_2) \tag{7}$$

for the temperatures of the condensed phases and

$$f = \frac{N_0}{(2\pi RT_0)^{3/2}} \exp\left\{ -\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_0} \right\} \qquad (0 < x < L) \tag{8}$$

for the distribution function of the molecules of the gas phase, the components of the molecular velocity vector (ξ_x, ξ_y, ξ_z) having all possible values. The boundary conditions at the edges of the two condensed phases and at the interface surfaces between the gas phase and the condensed phases become as follows:

$$\left. \begin{array}{ll} \tilde{T}_1 = T_{c1} & \text{at} \quad x = -D_1 \\ \tilde{T}_2 = T_{c2} & \text{at} \quad x = L + D_2 \end{array} \right\} \quad \text{and} \quad \left. \begin{array}{ll} \tilde{T}_1 = T_{W1} & \text{at} \quad x = 0 \\ \tilde{T}_2 = T_{W2} & \text{at} \quad x = L \end{array} \right\} \tag{9}$$

and

$$f = \frac{\tilde{N}_{W1}}{(2\pi RT_{W1})^{3/2}} \exp\left\{ -\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_{W1}} \right\} \qquad \text{for} \quad \xi_x > 0 \qquad \text{at} \quad x = 0 \tag{10}$$

$$f = \frac{\tilde{N}_{W2}}{(2\pi RT_{W2})^{3/2}} \exp\left\{ -\frac{\xi_x^2 + \xi_y^2 + \xi_z^2}{2RT_{W2}} \right\} \qquad \text{for} \quad \xi_x < 0 \qquad \text{at} \quad x = L \tag{11}$$

for molecules leaving each of the interface surfaces, respectively, where T_{Wi}, which is an unknown parameter, represents the temperature of the interface surface itself and will eventually be determined as part of the solution. $\tilde{N}_{Wi}$, which is given by

$$\tilde{N}_{Wi} = \alpha_c N_{Wi} + (1 - \alpha_c) N_{Di} \qquad (0 \leqq \alpha_c \leqq 1) \tag{12}$$

is the number density for those molecules emitted from the interface, α_c being a parameter associated with a certain kind of the imperfectness of the interface (see [5]) and is called the *imperfectness* parameter here. N_{Wi} is the saturated vapor number density at the temperature T_{Wi} of the surface of the condensed phase to be determined by the Clapeyron-Clausius relation as

$$N_{Wi} = \frac{P_{Wi}}{k\, T_{Wi}}, \qquad \frac{P_{Wi}}{P_0} = \exp\left\{ -\Gamma\left(\frac{T_0}{T_{Wi}} - 1 \right) \right\} \qquad \left(\Gamma \equiv \frac{h_L}{RT_0} \right) \tag{13}$$

where Γ is a non-dimensional parameter defined by $\Gamma \equiv h_L/(RT_0)$, h_L being the latent heat of vaporization per unit mass. The other quantity N_{Di}, on the other hand, is the number density for molecules reflected from the condensed phase determined by the condition of *no net mass flow* across the phase boundary. N_{Di}, therefore, is obtained from the distribution function for incident molecules to the condensed phase as

$$N_{Di} = \mp \frac{2\sqrt{\pi}}{(2RT_{Wi})^{1/2}} \iiint_{\xi_x \lessgtr 0} \xi_x f \, d\xi_x d\xi_y d\xi_z \tag{14}$$

the upper sign applying to the interface at $x = 0$ and the lower to that at $x = L$, respectively. It may be noted that N_{Di} is also an unknown constant to be determined as part of the solution. In addition to the above specification of the conditions at the boundary surfaces, the condition of the continuity of energy flow across each of the interfaces has to be imposed, which may be written as

$$\mp \lambda_c \frac{\partial \tilde{T}_i}{\partial x} = \pm E \pm \rho\, u\, (h_L - h) \tag{15}$$

where λ_c is the thermal conductivity of the condensed phase itself (assumed constant). h is the enthalpy per unit mass of the gas and $h = c_p T$, c_p being the specific heat at constant pressure. E is the component of the energy flux of the gas in the x-direction defined by

$$E = \frac{1}{2} m \iiint \xi_x (\xi_x^2 + \xi_y^2 + \xi_z^2)\, f \, d\xi_x d\xi_y d\xi_z \tag{16}$$

which is here to be evaluated at the interfaces.

CHARACTERISTIC PARAMETERS

For the analysis of the present problem, we have introduced L as the length scale and τ_0 as the time scale taken as

$$\tau_0 \equiv \frac{L}{(2RT_0)^{1/2}} = \left(\frac{\gamma}{2}\right)^{1/2} \frac{L}{c_0} = \frac{1}{N_0 \nu_c} = \frac{\mu_0}{P_0} \tag{17}$$

where c_0, taken as the velocity scale here, is the sound speed of the gas at the initial state defined by $c_0 \equiv (\gamma RT_0)^{1/2}$, γ being the specific heat ratio ($\gamma = 5/3$ here). With the fluid dynamic quantities at the initial state together with these length, velocity and time scales, the system of the governing heat conduction and kinetic equations and the initial and boundary conditions is appropriately nondimensionalized, giving the following non-dimensional parameters characterizing the present flow field

$$\frac{T_{c1}}{T_0}, \quad \frac{T_{c2}}{T_0}, \quad \frac{D_1}{L}, \quad \frac{D_2}{L}, \quad \frac{\lambda_c}{\lambda_0}, \quad \frac{\kappa_c}{\kappa_0}, \quad \Gamma \equiv \frac{h_L}{RT_0}, \quad Kn \equiv \frac{l_0}{L}, \quad \alpha_c \tag{18}$$

where λ_0 and $\kappa_0 \equiv \lambda_0/(\rho_0 c_p)$ are the thermal conductivity and the thermal diffusivity of the gas at the initial state, respectively. Kn is the Knudsen number defined by $Kn \equiv l_0/L$, l_0 being the mean free path of the gas molecules at the initial state defined by $l_0 = (\mu_0/P_0)(8RT_0/\pi)^{1/2}$. It may be noted that the Reynolds number Re defined by $Re \equiv \rho_0 c_0 L/\mu_0$ is related here to the Knudsen number as $Re = (8\gamma/\pi)^{1/2}(1/Kn)$.

RESULTS

The system of the governing kinetic equations has been solved numerically subject to the initial and boundary conditions with a simple difference scheme applied. Here only a part of the results will be shown. A sample of the development of a transient flow field to its steady state is shown in Fig. 1 and 2, where the temperature jumps at the interfaces $x = 0$ and $x = L$, respectively, can clearly be visible at the final state ($t/\tau_0 = 120.0$ is considered to be the final steady state).
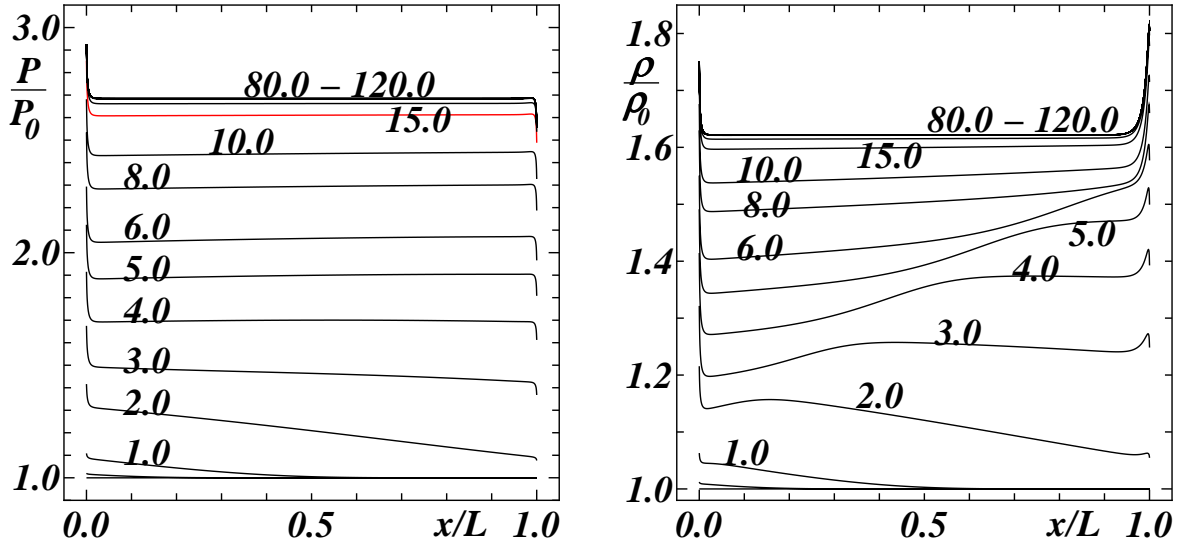
FIGURE 1. Transient distributions of the pressure and the density. The interface surfaces are at $x/L = 0$ and $x/L = 1$, respectively. $D_1/L = D_2/L = 0.1$, $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$, $\Gamma = 3$, $\lambda_c/\lambda_0 = 80.0$, $\kappa_c/\kappa_0 = 0.50$, $Kn = 0.005$ ($Re = 412.03$), $\alpha_c = 1.0$. The numbers in the figures indicate the time t/τ_0.

The effects of the latent heat parameter Γ coupled with the thermal conductivity ratio λ_c/λ_0 on the flows are virtually the same as in the half-space problem done by Onishi & Yamada [1], i.e., $\Gamma \to$ small or $\lambda_c/\lambda_0 \to$ large, the temperature gradient $\partial \tilde{T}/\partial x$ within the condensed phase can become small, which means the possible existence of fairly large temperature jump at the interface leading to the fairly strong phase change process taking place. The
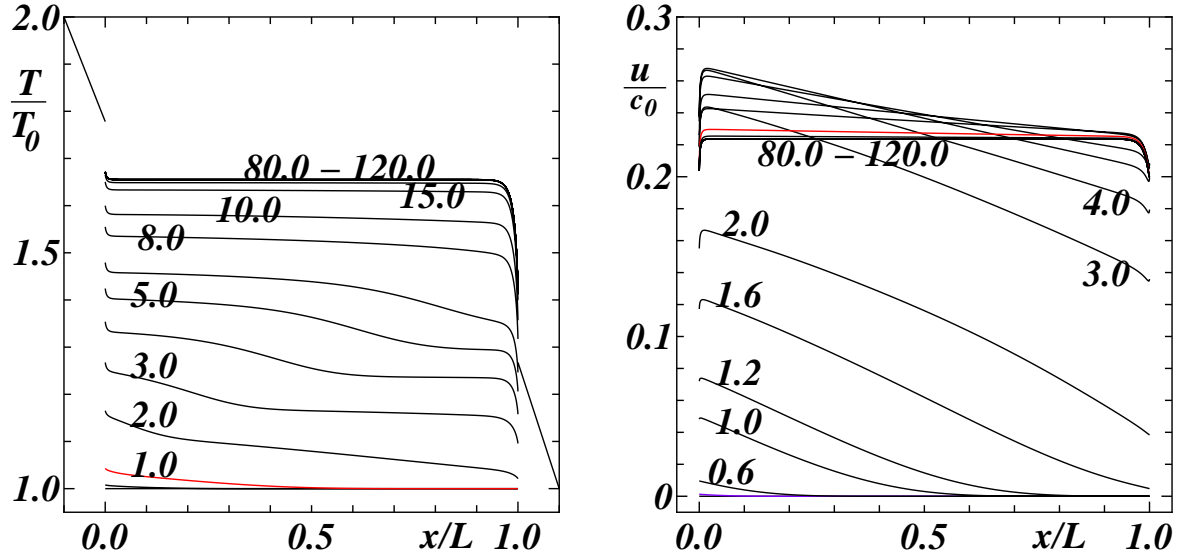
FIGURE 2. Transient distributions of the temperature and the velocity (continued on Fig. 1). The temperature profiles within the condensed phases are at $t/\tau_0 = 120.0$

mass and energy flows naturally become large as $\Gamma \to$ small. However, this applies down to a certain definite value of Γ; when Γ becomes smaller than this definite value, the mass and energy flows begin to decrease again. Figure 3 shows this behavior, indicating that the existence of the maxima of the mass and energy flows at a certain value of the parameter Γ. This will be a new feature among the roles of the latent heat parameter.

Another feature of the present problem is that the *negative mass flow* manifests itself during a certain period of time when the latent heat parameter Γ becomes extremely small but it disappears soon as time goes on. A typical example of this is shown in the graph on the right hand side of Fig. 4, where the transient mass flow distributions are also listed. The existence of this *negative mass flow* phenomenon, which has been noticed at the steady state and discussed by Sone & Onishi [4], may be explained as follows: consider the difference between the mass fluxes due to the molecules leaving the interfaces at $x = 0$ and $x = L$, respectively, which is in this case

$$\rho u|_{x=0} - \rho u|_{x=L} = \frac{1}{\sqrt{\pi}} \frac{P_0}{(2RT_0)^{1/2}} \left(\frac{\hat{P}_{W1}}{\sqrt{\hat{T}_{W1}}} - \frac{\hat{P}_{W2}}{\sqrt{\hat{T}_{W2}}} \right) \longrightarrow \text{negative for} \quad \hat{T}_{W1} > \hat{T}_{W2} \quad \text{for} \quad \hat{P}_{W1} \cong \hat{P}_{W2} \quad (19)$$

because the saturated vapor pressures $\hat{P}_{W1}$ and $\hat{P}_{W2}$ remain nearly equal even when $\hat{T}_{W2} \neq \hat{T}_{W1}$ for very small values of the latent heat parameter Γ (see Eq. (13)). Molecular collisions, of course, affect this difference in the mass flow. However, the above expression can be applied locally near the interface, in which case the interface at $x = L$ above, for example, becomes the position near the interface at $x = 0$, say, somewhere outside the Knudsen layer (temperature of the vapor near the interface at $x = 0$, of course, is lower than T_{W1} in the case considered).

Finally, as to the *negative temperature gradient* [3]-[4], a well-known phenomenon for two-surface problems, seems not to manifest itself in the present cases with internal structures.

ACKNOWLEDGMENTS

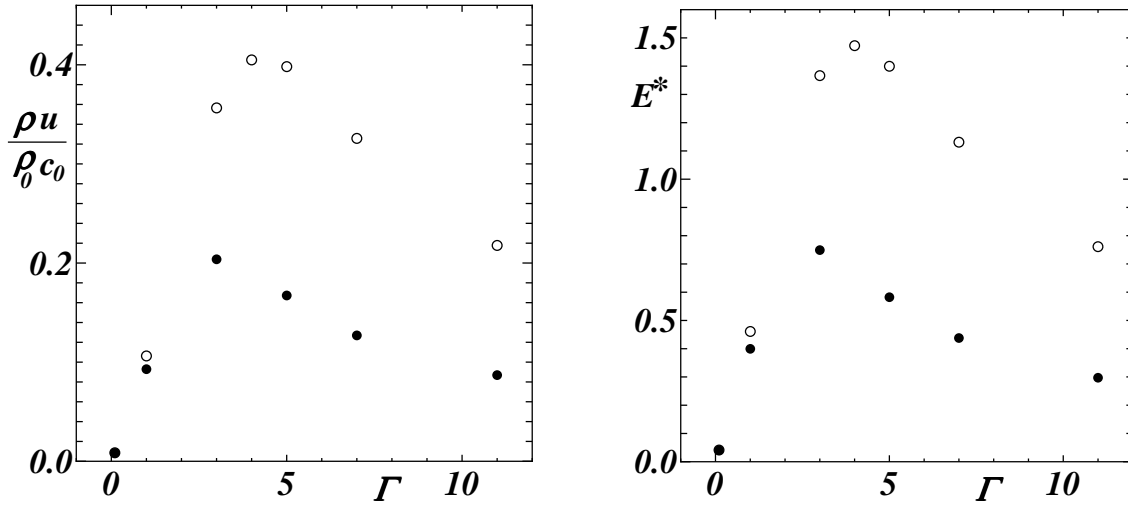**FIGURE 3.** The steady state mass flow $\rho u/\rho_0 c_0$ and energy flow $E^* = E/P_0(2RT_0)^{1/2}$ versus the latent heat parameter Γ. $D_1/L = D_2/L = 0.1$, $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$, $Kn = 0.005$ ($Re = 412.03$), $\alpha_c = 1.0$. The **white circle:** $\lambda_c/\lambda_0 = 80.0$, $\kappa_c/\kappa_0 = 0.50$; The **balck circle:** $\lambda_c/\lambda_0 = 30.0$, $\kappa_c/\kappa_0 = 0.32$. The existence of the maxima of the mass and energy flows can be expected.
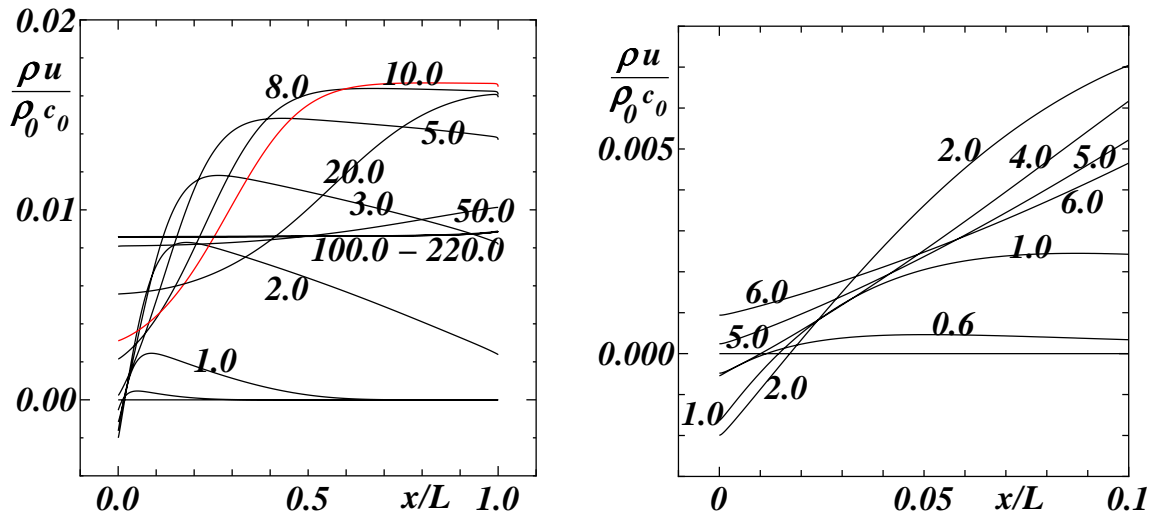

FIGURE 4. A transient to steady state mass flow distributions. $D_1/L = D_2/L = 0.1$, $T_{c1}/T_0 = 2.0$, $T_{c2}/T_0 = 1.0$, $\Gamma = 0.1$, $\lambda_c/\lambda_0 = 80.0$, $\kappa_c/\kappa_0 = 0.5$, $Kn = 0.005$ ($Re = 412.03$), $\alpha_c = 1.0$. The numbers in the figures indicate the time t/τ_0. The graph on the right, which is the enlarged portion of the region near the interface at $x/L = 0$, indicates the negative mass flow persisting up to about $t/\tau_0 = 4.5$.

REFERENCES

1. Onishi, Y. and Yamada, K., "Evaporation and Condensation from or onto the Condensed Phase with an Internal Structure." in *Rarefied Gas Dynamics* (AIP Conference Proceedings: Volume 663), edited by A.D. Ketsdever and E.P. Muntz, American Institute of Physics, New York, 2003, pp. 630–637.
2. Bhatnagar, P.L., Gross, E.P. and Krook, M., *Phys. Rev.* **94** (1954) 511–525.
3. Pao, Y.P., *Phys. Fluids* **14** (1971) 306–312.
4. Sone, Y. and Onishi, Y., *J. Phys. Soc. Japan* **44** (1978) 1981–1994.
5. Mager, R., Adomeit, G. and Wortberg, G., "Theoretical and experimental investigation of the strong evaporation of solids." *Rarefied Gas Dynamics: Physical Phenomena* (Progress in Astronautics and Aeronautics, Vol. 117), edited by E.P. Muntz, D.P. Weaver and D.H. Campbell, Washington D.C.: AIAA, 1989, pp. 460–469.